classmate

Date

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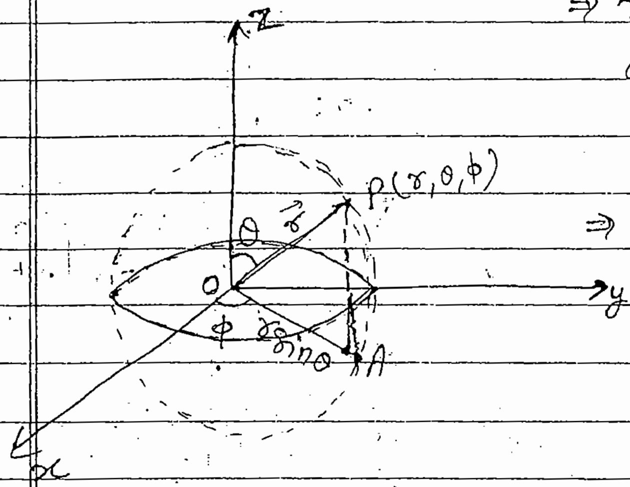
Spherical Polar Co-ordinate system :-



⇒ $\vec{r}$ is radial vector & is always measured from the centre of sphere. It'll be same & all dirⁿ.

⇒ θ is angle b/w $\vec{r}$ & $\hat{z}$

φ is angle b/w r sinθ & x axis.

$Z = r\cos\theta$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

Limits :-

r : 0 to ∞

$\theta : 0 \rightarrow \pi$

$\phi : 0 \rightarrow 2\pi$

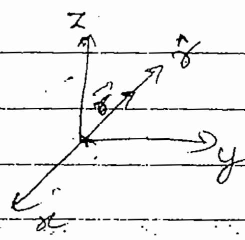
For Hemisphere , $\theta : 0 \rightarrow \pi/2$

$\phi : 0 \rightarrow 2\pi$

for 1/4 sphere , $\theta : 0 \rightarrow \pi/2$

$\phi : 0 \rightarrow \pi$

Length Elements :-



$\vec{dl_r} = dr\,\hat{r}$

$\hat{x} \rightarrow \hat{y} \rightarrow \hat{z} \rightarrow \hat{x}$

$\hat{x} \times \hat{y} = \hat{z}$

$\hat{x}\cdot\hat{x} = 1$, $\hat{x}\cdot\hat{y} = 0$

$\hat{r} \rightarrow \hat{\theta} \rightarrow \hat{\phi} \rightarrow \hat{r}$

$\hat{r}\cdot\hat{r} = 1$

$\hat{r}\cdot\hat{\theta} = 0$

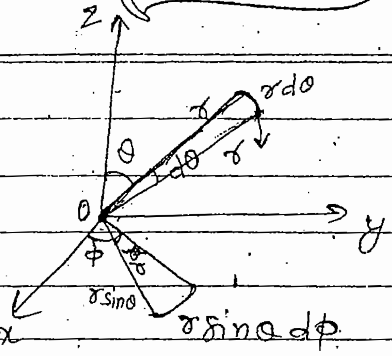
$\hat{r} \times \hat{\theta} \Rightarrow \hat{\phi}$

$\hat{r}\cdot\hat{\phi} = 0$

dirⁿ of $\hat{r}$ is always away from the origin.

$+\hat{r}$ → going away from origin

$-\hat{r}$ → coming towards the origin



$\vec{dl_\theta} = r\,d\theta\,\hat{\theta}$ (dir^n of change clockwise)

$\vec{dl_\phi} = r\sin\theta\,d\phi\,\hat{\phi}$

(dir^n of change anticlockwise viewing from +ve z dir^n)

Surface Elements :- 3 surfaces.. $r\theta$, $\theta\phi$, $r\phi$.

$\vec{dS_r} \rightarrow$ surface which does not change in r. r will be fix.

$\hat{r}$ always normal to surface

Outer spherical surface is $\vec{dS_r}$

$\vec{dS_r} = r^2 \sin\theta\,d\theta\,d\phi\,\hat{r}$ (Sur. area $= 4\pi r^2$)

$\vec{dS_\theta} \rightarrow$ surface where θ will not change.

2 dim. version of sphere is disc, for a disc, θ will not change.

$\vec{dS_\theta} = r\sin\theta\,dr\,d\phi\,\hat{\theta}$

$= r\,dr\,d\phi\,\hat{\theta}$

$\theta = 90^\circ$ for disc
$r \rightarrow 0$ to r
$\phi \rightarrow 0$ to 2π
& get $dS_\theta = \pi r^2$

$$\int_0^r r\,dr \int_0^{2\pi} d\phi$$

$$\frac{r^2}{2} \times 2\pi = \pi r^2 \text{ (surface area)}$$

$\vec{dS_\phi} = r\,dr\,d\theta\,\hat{\phi}$ (least used)

Volume Element

$\therefore d\tau = \vec{dl_r} \cdot (\vec{dl_\theta} \times \vec{dl_\phi})$

$d\tau = r^2 \sin\theta\,d\theta\,d\phi\,dr$

Que:- Calculate the volume of a sphere of radius R.

$$\tau = \int_0^R \int_0^\pi \int_0^{2\pi} d\tau$$

$$\tau = \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= \left[\frac{r^3}{3}\right]_0^R \left[-\cos\theta\right]_0^\pi \left[\phi\right]_0^{2\pi}$$

$$= \frac{R^3}{3} \left[1+1\right]\left[2\pi\right]$$

$$= \frac{4}{3}\pi R^3$$

Relation b/w Unit Vectors :-

$\hat{r} \quad \hat{\theta} \quad \hat{\phi}$

$\downarrow$

$\hat{x} \quad \hat{y} \quad \hat{z}$

$z = r\cos\theta$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$$\boxed{r = \sqrt{x^2+y^2+z^2}} = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi}$$

$$= r\sqrt{\cos^2\theta + \sin^2\theta(\cos^2\phi + \sin^2\phi)}$$

$$= r\sqrt{\cos^2\theta + \sin^2\theta}$$

$$= r$$

$$\theta = \cos^{-1}\frac{z}{r} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)}$$

$\phi = ?$

$$\frac{y}{x} = \tan\phi$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{y}{x}\right)}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\boxed{\vec{r} = r\sin\theta\cos\phi\,\hat{x} + r\sin\theta\sin\phi\,\hat{y} + r\cos\theta\,\hat{z}}$$

This is the func^n of 3 variables r, θ, ϕ.

$\hat{r} \rightarrow$ dir^n of increment of r

$\hat{\theta} \rightarrow$ " " θ

$\hat{\phi} \rightarrow$ " " ϕ

$$d\vec{r} = \frac{\partial \vec{r}}{\partial r} dr + \frac{\partial \vec{r}}{\partial \theta} d\theta + \frac{\partial \vec{r}}{\partial \phi} d\phi$$

$$\vec{r} = r\hat{r}$$

$$\hat{r} = \frac{\frac{\partial \vec{r}}{\partial r}}{\left|\frac{\partial \vec{r}}{\partial r}\right|}$$

$$\frac{\partial \vec{r}}{\partial r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$

$$\left|\frac{\partial \vec{r}}{\partial r}\right| = \sqrt{\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta}$$

$$= \sqrt{\sin^2\theta(\cos^2\phi + \sin^2\phi) + \cos^2\theta}$$

$$= 1$$

$$\hat{r} = \frac{\partial \vec{r}}{\partial r} \Rightarrow \boxed{\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}} \quad \text{— (1)}$$

$$\hat{\theta} = \frac{\frac{\partial \vec{r}}{\partial \theta}}{\left|\frac{\partial \vec{r}}{\partial \theta}\right|}$$

$$\frac{\partial \vec{r}}{\partial \theta} = r\cos\theta\cos\phi\,\hat{x} + r\cos\theta\sin\phi\,\hat{y} - r\sin\theta\,\hat{z}$$

$$\left|\frac{\partial \vec{r}}{\partial \theta}\right| = \sqrt{r^2\cos^2\theta\cos^2\phi + r^2\cos^2\theta\sin^2\phi + r^2\sin^2\theta}$$

$$= r\sqrt{\cos^2\theta(\cos^2\phi + \sin^2\phi) + \sin^2\theta}$$

$$= r$$

$$\boxed{\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}} \quad \text{— (2)}$$

$$\hat{\phi} = \frac{\frac{\partial \vec{r}}{\partial \phi}}{\left|\frac{\partial \vec{r}}{\partial \phi}\right|}$$

$$\frac{\partial \vec{r}}{\partial \phi} = -r\sin\theta\sin\phi\,\hat{x} + r\sin\theta\cos\phi\,\hat{y}$$

$$\left|\frac{\partial \vec{r}}{\partial \phi}\right| = \sqrt{r^2\sin^2\theta\sin^2\phi + r^2\sin^2\theta\cos^2\phi}$$

$$= r\sqrt{\sin^2\theta(1)}$$

$$= r\sin\theta$$

$$\boxed{\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}} \quad \text{—— (3)}$$

Using these ~~(3)~~ (1), (2), (3) eqⁿ convert spherical to cartisian.

eqⁿ (1) × sin θ, (2) × cos θ } add

$$\sin\theta\,\hat{r} = \sin^2\theta\cos\phi\,\hat{x} + \sin^2\theta\sin\phi\,\hat{y} + \cos\theta\sin\theta\,\hat{z}$$

$$\cos\theta\,\hat{\theta} = \cos^2\theta\cos\phi\,\hat{x} + \cos^2\theta\sin\phi\,\hat{y} - \cos\theta\sin\theta\,\hat{z}$$

$$\sin\theta\,\hat{r} + \cos\theta\,\hat{\theta} = \cos\phi\,\hat{x} + \sin^2\phi\,\hat{y} \quad \text{—— (4)}$$

eqⁿ (4) × cos φ − (3) × sin φ,

$$\boxed{\sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi} = \hat{x}} \quad \text{—— (5)}$$

(4) × sin φ + (3) × cos φ

$$\boxed{\sin\theta\sin\phi\,\hat{r} + \sin\phi\cos\theta\,\hat{\theta} + \cos\phi\,\hat{\phi} = \hat{y}} \quad \text{—— (6)}$$

(1) × cos θ − (2) × sin θ

$$\cos\theta\sin\theta\cos\phi\,\hat{x} + \cos\theta\sin\theta\sin\phi\,\hat{y} + \cos^2\theta\,\hat{z} - \sin\theta\cos\theta\cos\phi\,\hat{x}$$
$$+ \sin\theta\cos\theta\sin\phi\,\hat{y} - \sin^2\theta\,\hat{z} = \hat{z}$$

$$\boxed{\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}} \quad \text{—— (7)}$$

$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$
$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$
$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$

$\hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$
$\hat{y} = \sin\theta\sin\phi\,\hat{r} + \sin\phi\cos\theta\,\hat{\theta} + \cos\phi\,\hat{\phi}$
$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

Cylendrical Co-ordinate system :-

s, ϕ, z are mutually orthogonal to eachother.

$\vec{s}$ is radius vector of the cylender.

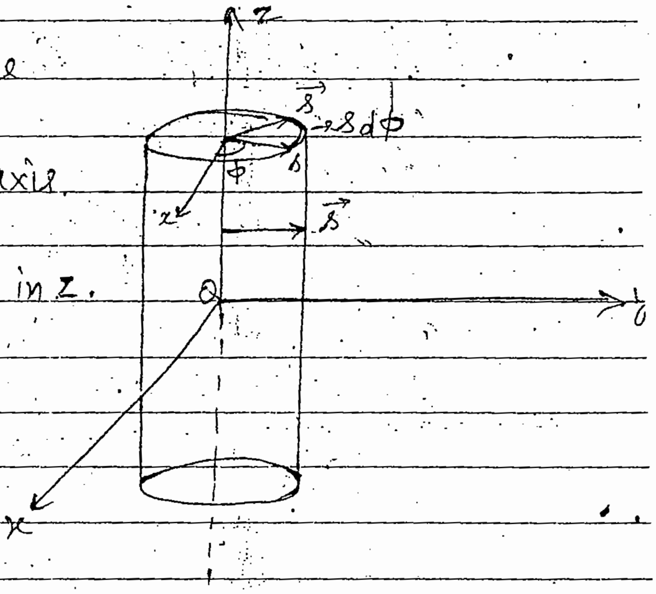
$\phi \rightarrow$ angle b/w $\vec{s}$ & x-axis.

$z = z$. It is length vector. There is no change in z.

$x = s\cos\phi$

$y = s\sin\phi$

$z = z$



radius vector $\leftarrow \vec{s} = s\,\hat{s} \rightarrow$ unit vector

$\hat{s}$ will be +ve if move away from axis
" " −ve " " toward the axis.
$\hat{s}$ is the normal to the curved surface & +ve

Total Surface area $= \underset{\text{cross sec.}}{\pi r^2} + \underset{\text{cylendrical curved}}{2\pi r l}$

Limits :-
$s : 0 \rightarrow \infty$
$\phi : 0 \rightarrow 2\pi$
$z : -\infty \rightarrow +\infty$